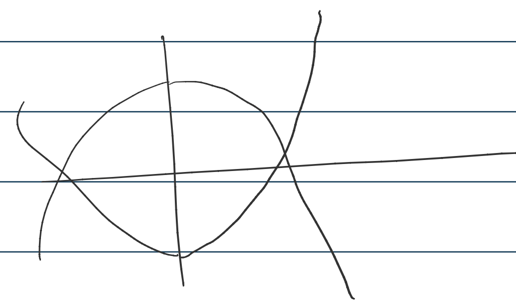
Math 53, Discussion 105

Thursday, March 19, 2020 5:51 PM

1) Evaluate $\iint_D x(y-1)\,dA$ where D is region bounded by $y = 1 - x^2$ and $y = x^2 - 3$, answer should be 0
2) Evaluate $\iint_D \sqrt{1+4x^2+4y^2}\,dA$ where D is the bottom half of $x^2 + y^2 = 16$, answer: $\frac{1}{12}\pi\left(65^{\frac{3}{2}} - 1\right)$
3) Evaluate $\iiint_E 6z^2\,dV$ where E is the region below $4x + y + 2z = 10$ in the first octant. Answer: $\frac{625}{2}$

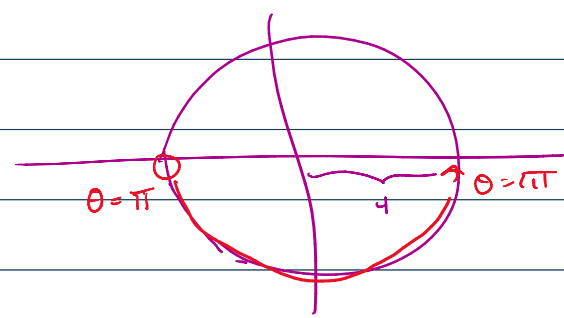
1) $\displaystyle\int_{-\sqrt{2}}^{\sqrt{2}}\int_{x^2-3}^{1-x^2} x(y-1)\,dy\,dx$



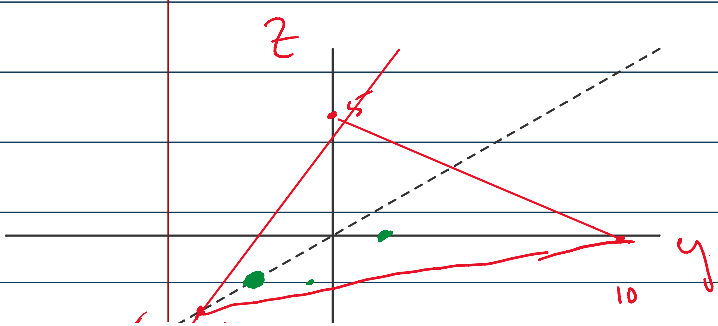
Compute...

2) use polar. integrand becomes $\sqrt{1+4r^2}\,r\,dr\,d\theta$

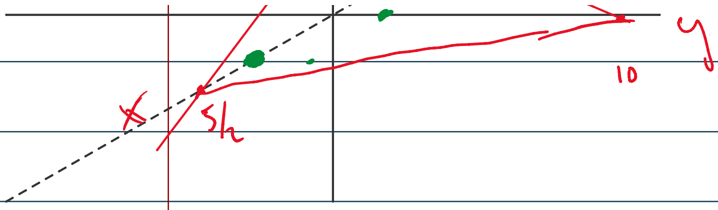
$\displaystyle\int_{\pi}^{2\pi}\int_0^4 \sqrt{1+4r^2}\; r\,dr\,d\theta$



3) $4x + y + 2z = 10$



$\displaystyle\int_0^{\frac{5}{2}}\int_0^{10-4x}\int_0^{5-\frac{y}{2}-2x} 6z^2\,dz\,dy\,dx$



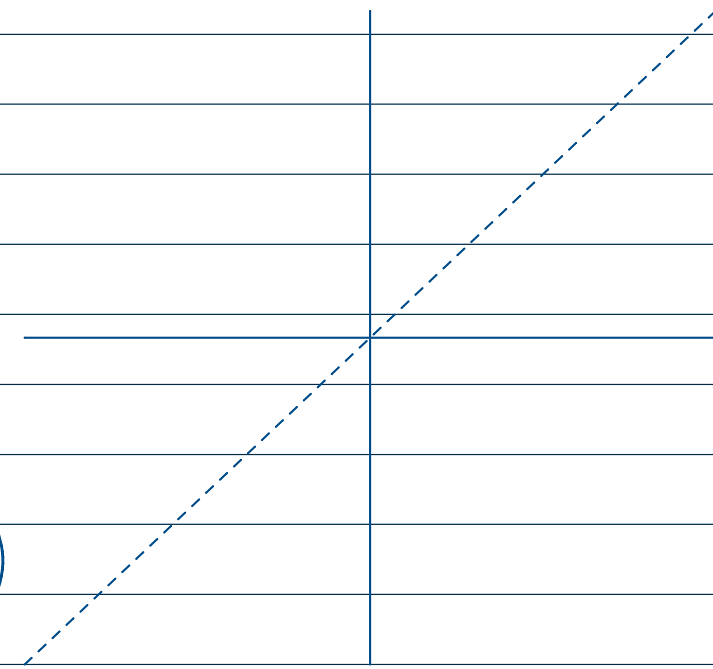
Region $= \{ (x,y,z)$ s.t. $0 \le x \le 5/2$

$0 \le y \le 10 - 4x$

$0 \le z \le 5 - \frac{y}{2} - 2x \}$

Could write this integral 6 ways (permutations of dx, dy & dz)

1. Evaluate $\iiint_E e^{-x^2-z^2}\,dV$ with E the region between the cylinders $x^2+z^2=4$ and $x^2+z^2=9$ with $1 \le y \le 5$ and $z \le 0$, answer: $2\pi(e^{-4}-e^{-9})$
2. Compute $\int_{-\infty}^{\infty} e^{-x^2}\,dx$ (hint: compute $\left(\int_{-\infty}^{\infty} e^{-x^2}\,dx\right)^2$ using polar coordinates)



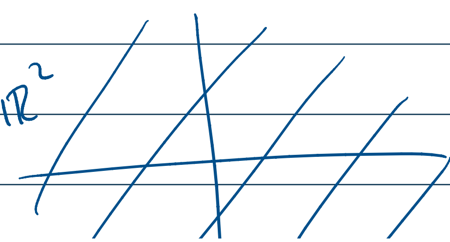
2) Let $I = \int_{-\infty}^{\infty} e^{-x^2}\,dx$

$$I^2 = \left(\int_{-\infty}^{\infty} e^{-x^2}\,dx\right)\left(\int_{-\infty}^{\infty} e^{-y^2}\,dy\right)$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-x^2} e^{-y^2}\,dx\,dy$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-x^2-y^2}\,dx\,dy$$

$D = \mathbb{R}^2$



$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-x^2-y^2}\,dx\,dy$$

Polar coordinate. $I^2 = \int_0^{2\pi}\int_0^{\infty} e^{-r^2}\, r\, dr\, d\theta$

$$= 2\pi \int_0^{\infty} e^{-r^2}\, r\, dr$$

$$= 2\pi \left[-\frac{1}{2} e^{-r^2} \Big|_0^{\infty} \right]$$

$$I^2 = -\frac{2\pi}{2}(0-1) = \pi$$

$$\Rightarrow I = \sqrt{\pi} = \int_{-\infty}^{\infty} e^{-x^2}\,dx$$